School of Electrical Engineering
Electronic Systems Research Laboratory
Purdue University
Lafayette, Indiana

STUDY OF LUNAR REFLECTIVE COMPONENTS OF SOLAR EMISSION

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G. R. Cooper
C. D. McGillem

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1. INTRODUCTION

The unsuitability of solar radio emission as a signal source for studying scattering from the lunar surface with a wideband crosscorrelator * has led to an investigation of the use of an active system in which a locally generated signal is transmitted and the scattered radiation crosscorrelated with the transmitted signal. The signal which has been under study is bandlimited "white noise." This type of signal has a number of advantages: among them are the availability of a noise-free reference and the possibility of very high duration-bandwidth products thereby making possible good resolution in both range and velocity.

During the reporting period a preliminary analysis was made of the use of an active coherent random signal radar for experimental studies of lunar and planetary surfaces. Also, an analysis was completed on the electromagnetic impulse response of various scattering bodies thereby making this theoretical tool more useful in the analysis of performance of random signal radars (or any other types of radars) against various targets. Correlator development has continued and the designs completed to permit experimental evaluation of the random signal radar. The status of research on the above areas is summarized in the following sections of this report.

* Study of Lunar Reflective Components of Solar Emission, Semi-Annual Status Report, Feb. 1 to July 31, 1965, NsG-543, Purdue University, Lafayette, Indiana.

2. MEASUREMENT OF LUNAR AND PLANETARY SURFACE CHARACTERISTICS

Measurement Concept

The wideband digital correlator makes it possible to measure crosscorrelation functions which are extremely narrow. This capability can be utilized to implement an active, wideband, random signal radar system which exhibits extraordinarily good resolution in range measurements without introducing the doppler ambiguities associated with pulse-type radars. This range resolution creates the possibility of measuring certain characteristics of the lunar surface (or any planetary surface) which are not readily obtainable by other means.

Two measurement possibilities are of particular interest in connection with the lunar surface. One is the possibility of determining the depth of a possible dust layer by using a random signal radar located on a satellite or on the earth. The second is the possibility of using such a radar for measuring surface roughness.

The determination of dust layer depth is based on the assumption that signal reflections will occur at both the top and bottom of the layer and that the radar has sufficient range resolution to be able to resolve these two. The measurement of surface roughness assumes that individual prominences and depressions on the surface will possess specular reflecting points whose distribution in range can be determined from the spread of the crosscorrelation function. In order to assess more accorately the feasibility of making either type of measurement, a brief review of the theory of random signal radars will be presented.

Theory of Random Signal Radars

It is assumed that the transmitted radar signal, $x(t)$, is a sample function from a wideband stationary random process having an autocorrelation function of $R_x(\tau)$. The reflected signal from any specular reflecting point will be of the same form but will be delayed in time and will also have a modified time scale if there is relative motion between the radar transmitter and the reflecting point. In particular, the k^{th} such reflected signal may be represented as $a_k x(\beta_k t - \tau_k)$ where τ_k is the delay at $t = 0$, a_k is an amplitude factor, and

$$\beta_k \doteq \frac{c + v_k}{c - v_k}$$

where v_k is the radial velocity of the reflecting point (toward the radar) and c is the electromagnetic wave velocity. If there are N such reflecting points at different ranges, and possibly different velocities, the total returned signal is

$$y(t) = \sum_{k=1}^{N} a_k x(\beta_k t - \tau_k).$$

The crosscorrelation function of the transmitted signal and the total returned signal may be written as

$$R_{yx}(\tau) = E\left[y(t)x(t-\tau)\right] = \sum_{k=1}^{N} a_k R_x\left[\tau - \tau_k + (\beta_k - 1)t\right]$$

The output of the crosscorrelator is an estimate of this crosscorrelation for the value of delay set into the crosscorrelator. If this delay is made to vary linearly with time so that

$$\tau = \tau_o - (\beta_o - 1)t$$

then this output has a mean value of

$$Z(\beta_o, \tau_o) = R_{yx}\left[\tau_o - (\beta_o - 1)t\right]$$

$$= \sum_{k=1}^{N} \alpha_k R_x\left[(\beta_k - \beta_o)t - (\tau_k - \tau_o)\right]$$

Except for the case in which β_o equals some β_k, $Z(\beta_o, \tau_o)$ is an oscillating function of time and will average to zero. When $\beta_o = \beta_k$, adjustment of τ_o can be made to obtain a measurement of τ_k when an envelope correlator is used.

For the applications being considered here, however, the envelope correlator is not used and β_o is not made equal to any β_k. Hence, it is desirable to consider in more detail the manner in which the correlator output varies with time. In order to do this, assume that the spectral density of x(t) is symmetrical about some high frequency, ω_c. The autocorrelation function of x(t) can then be expressed as

$$R_x(\tau) = R_c(\tau) \cos \omega_c \tau$$

where $R_c(\tau)$ is the <u>envelope</u> of the correlation function and depends upon the bandwidth of the transmitted signal but not upon its center frequency. The correlator output may now be written as

$$Z\,(\beta_o, \tau_o) = \sum_{k=1}^{N} \alpha_k \, R_c\left[(\beta_k - \beta_o)t - (\tau_k - \tau_o)\right] \operatorname{Cos} \omega_c\left[(\beta_k - \beta_o)t - (\tau_k - \tau_o)\right]$$

Each term of this expression is also oscillatory but at a much lower frequency. The manner in which the k^{th} component of $Z(\beta_o, \tau_o)$ might vary with time is illustrated in Fig. 1.

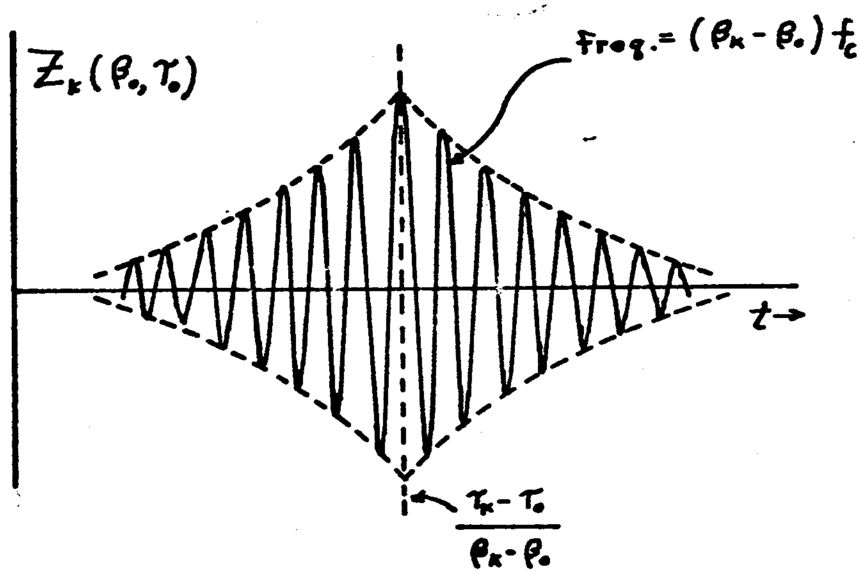


Fig. 1. One component of correlator output.

The components in a given frequency range can be extracted by means of a narrow-band filter at the output of the correlator. The other components will average to zero. Hence, it is possible to separate out reflections having different doppler shifts even though they may be at the same range.

It may also be noted that the correlator output for a single reflection is a replica of the autocorrelation function of the transmitted signal, but is stretched out in time by the factor $\frac{1}{\beta_k - \beta_o}$. The resulting pulse must be much longer than the smoothing time of the correlator in order to avoid excessive noise in the output. This implies that $(\beta_k - \beta_o)$ must be a very small number.

As an illustration, suppose that $R_x(\tau)$ has an effective width of 10 ns (i.e., the transmitted signal has a bandwidth of about 100 Mc) and a center frequency of 10 Gc. If the correlator sampling rate is 1 Mc and 10^4 samples are required to reduce the noise adequately, then the smoothing time is about .01 second. Hence, a value of $(\beta_k - \beta_o)$ which stretches the correlation function out to 0.1 second should be adequate. Thus,

$$\beta_k - \beta_o \approx \frac{10 \times 10^{-9}}{0.1} = 10^{-7}$$

If the relative velocity of the reflecting point is 1 mile/second, then

$$\beta_k \approx 1 + 10^{-5}$$

and

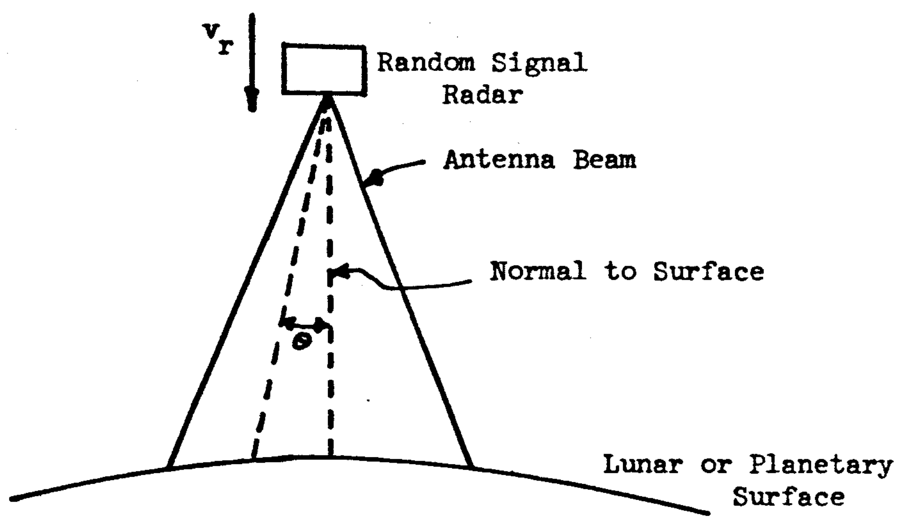
$$\beta_o - 1 \approx (\beta_k - 1) - 10^{-7} = 10^{-5} - 10^{-7}$$

Since $(\beta_o - 1)$ is the delay-rate provided in the correlator, this result implies that the delay must change at a rate of 10 μsec. per second. This is equivalent to changing the sampling rate on the reflected signal by 10 cps. It may also be noted the center frequency of the correlator output for this component is 1,000 cps and the bandwidth required in a bandpass filter to provide adequate smoothing is 100 cps.

The above calculations serve to illustrate that the required bandwidths, delay-rates, etc., are not unreasonable and can be achieved in a practical system without difficulty. If the relative velocity of the reflecting point were much less than 1 mile/second, the only thing which would change would be the delay-rate in the correlator and there is no fundamental difficulty in making this quantity as small as desired.

Measurement of Dust Layer Thickness

In order to carry out the measurement of dust layer thickness, it is proposed that the antenna beam from a random signal radar can be directed toward the lunar or planetary surface as shown in Fig. 2.



Feb. 2. Geometry of measurement method.

The relative velocity of the radar along the normal to the surface is designated as v_r. Hence, this is also the relative velocity of reflecting points near the center of the beam. Points off the center of the beam at an angle θ will have a relative velocity of $v_r \cos \theta$. Because of this difference in velocity it is possible to make the effective beam width smaller by integrating the correlator output for longer periods of time. This point will be considered in more detail later.

The basic assumption involved in the proposed method of measurement is that separate and resolvable reflections occur at both the top and bottom of the dust layer. There is little doubt that a reflection will occur at the upper surface, which is an interface between dust and space. It is also reasonable to assume that a smaller, but still significant, reflection will occur at the interface between the dust and the solid material below.

Another assumption which is implicit in this method is that the attenuation in the dust layer is sufficiently low that the second reflection has an observable amplitude after penetrating the layer twice. Just how low this needs to be depends upon the available signal power, the width and shape of the signal correlation function, and the added noise in the system.

In the simplest situation, the measurement would consist of measuring the relative delay between the two reflections which have been assumed. In a practical case, however, the situation is not likely to be this simple. For example, it is reasonable to expect that there will be a multitude of specular reflections from peaks and valleys which have no relation to the dust layer. Hence, it would be very difficult to select the pair of returns which would yield the desired result. Even in the absence of other specular points, there might well be multiple reflections in the dust layer which would create additional responses.

There are several ways in which the appropriate reflections might possibly be separated from the unwanted ones. In the first place, a specular reflection from a curved surface is probably much more sensitive to the polarization of the incident wave than a reflection from a smooth dust layer. Thus, the polarization could be changed periodically and the pair of reflections exhibiting the least variation taken to be the proper one. A similar result might occur as the angle to the specular points changes as a consequence of radar motion. Finally, the area being examined can be made smaller by increasing the smoothing time of the correlator thereby making the effective beamwidth smaller.

The effective beamwidth is determined by the velocity component of the radar, v_r, the center frequency, and the smoothing time of the correlator. The difference in doppler frequency between a point on the beam axis and a point off the axis by an angle θ is

$$\left| (\beta_r - \beta_o)\, f_c \right| \approx \frac{2\ {}_{r}}{c} \quad (1 - \operatorname{Cos} \theta)\, f_c$$

As an example, let v_r = 0.01 mile/sec and consider a frequency of 10 Gc and smoothing time of 10 seconds. The angle for which frequency components are less than 0.1 cps, will be on the order of 1 degree. This can be made much smaller by increasing the smoothing time. It will also be smaller for larger values of v_r or for a higher transmitted frequency, f_c.

The range resolution required to carry out effective measurements of the dust layer thickness must be extremely good - probably on the order of a few inches. Hence, the bandwidth of the random transmitted signal must be on the order of 1 Gc. While this bandwidth has not yet been achieved, it does not seem to be unreasonable.

Measurement of Surface Roughness

The geometrical configuration of Fig. 2 also applies to the measurement of surface roughness. In this case, however, it is much easier to extract the desired information because all reflections are useful. As noted previously, a multitude of specular reflections will result from the various prominences and depressions on the surface. Each of these reflections will produce a correlator output as shown in Fig. 1, but they will occur at times determined by the elevation of the reflecting point and at frequencies determined by the angle of that point with respect to the beam axis. The smoothing time of the correlator will determine the effective beamwidth and, hence, the area being examined on the surface.

The surface roughness will be related to the time distribution of the individual components of the reflected signal. There are a variety of ways that this information might be presented. One way would be to present the distribution or density function of the surface elevation as indicated by a power vs. time plot of the correlator output. Another method is to estimate the variance of the surface elevation and present a single number as a measure of surface roughness. A third method would be to observe the lowest and highest points and thus use the change in elevation as a measure.

3. ELECTROMAGNETIC IMPULSE RESPONSE

Significant progress has been made with the problem of relating the impulse response of scattering bodies to the details of their shapes and compositions. The initial object of this work was to infer some characteristics of the lunar surface from the nature of the scattered fields - particularly scattered noise fields. However, the results are more far-reaching than this. In order to characterize certain types of surface features, a study was made of the impulse responses of several simple shapes. Then, more complicated scatterers were synthesized by fitting together these simple shapes into various composites.

Except for a few special shapes, such as the sphere and cylinder, it is probably not possible to calculate an impulse response that is exact in any sense. The approximations which can be made generally fall into one or the other of the categories, high frequency approximations or low frequency approximations. For the most part, the high frequency approximations have the more interesting features.

To systematize the calculations, to point up exactly which approximation is being used and to show that approximation may be improved, it is helpful to calculate the impulse response of scatterers by the use of the generalized time domain reciprocity theorem. A form of this equivalent to those published by Cheo, * was obtained earlier. The scattered fields are calculated from approximations to the current distributions which are established on the scatterer by the incident field.

A summary of the results of the impulse response calculations for simple shapes will now be given.

1. Backscatter from small spheres of radius a. (Low frequency approximation)

a. Conducting Sphere

$$E_Z (r_o,t) = \frac{\mu_o \epsilon_o \alpha^3}{2\pi r_o} \delta'' \left(t - \frac{2 r_o}{c}\right)$$

b. Dielectric Sphere (dielectric constant ϵ_1)

$$E_Z (r_o,t) = \frac{\mu_o \epsilon_o \alpha^3}{2\pi r_o} \frac{\epsilon_1 - \epsilon_o}{\epsilon_1 + 2\epsilon_o} \delta'' \left(t - \frac{2 r_o}{c}\right)$$

All results to follow are high frequency approximations.

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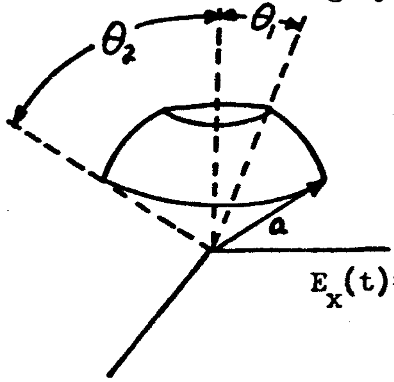
* Cheo, B. R., IEEE Trans. AP-13, p. 278, March 1965

2. Backscattering by a portion of a Conducting Spherical Surface



Center of spherical surface at the origin, scattering observed at distant point z.

$$E_x(t) = -\frac{c}{2\pi \epsilon_o}\left[\frac{\alpha}{2c}\left\{ \cos\theta_1 \delta\left(t - \frac{2(Z_o - \alpha \cos\theta_1)}{c}\right) - \cos\theta_2\, \delta\left(t - \frac{2(Z_o - \alpha \cos\theta_2)}{c}\right)\right\} + \frac{1}{4}\left\{ U\left(t - \frac{2(Z_o - \alpha\, \text{Cox}\, \theta_2)}{c}\right) - U\left(t - \frac{2(Z_o - \alpha \cos\theta_1)}{c}\right)\right\}\right]$$

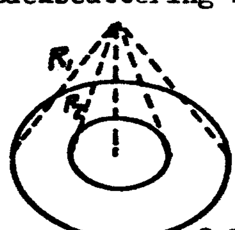
3. Backscattering by an Conducting Surface of Constant Delay.

Here we are concerned with any surface, in any medium, such that the propagation time to and from all points on the scatterer is the same. The area of the scatterer is A, R_o is the range, incident field starts out with a spherical wavefront:

$$E_x(t) = \frac{A}{4\pi^2 c R_o^2}\, \delta'(t - 2T)$$

where T is the constant delay time. Note that the impulse response for this type of surface is a doublet.

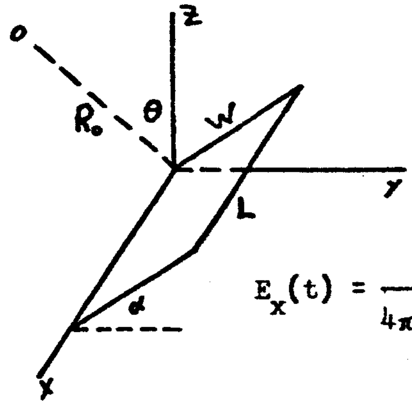
4. Backscattering by a plane conducting disk with hole



If the disk is small, and a long way from the primary source, to a front approximation we have plane waves on a plane surface so the previous result (3) applies. For larger disks, the result is

$$E_x(t) = \frac{c}{2R_o}\delta\left(t - \frac{2R_o}{c}\right) - \frac{c}{2R_1}\delta\left(t - \frac{2R_1}{c}\right) + \frac{1}{t^2}\left[U\left(t - \frac{2R_o}{c}\right) - U\left(t - \frac{2R_1}{c}\right)\right]$$

5. Scattering by a Rectangular Plate.



Plane wave comes in along z, with E along x. Observation point, O, is in y z plane at angle θ with z axis. Plate is of width W and length L and is inclined at an angle α to x y plane.

$$E_x(t) = \frac{W}{4\pi^2 R_o} \; \frac{\text{Cos}\alpha}{\text{Sin}\alpha + \text{Sin}(\alpha-\theta)} \left[\delta\left(t - \frac{Z_o + R_o - L\left[\text{Sin}\alpha + \text{Sin}(\alpha - \theta)\right]}{}\right) - \delta\left(t - \frac{Z_o + R_o}{c}\right) \right]$$

where Z_o is distance to plane wave source.

6. Backscattering by a Section of Conducting Conical Surface.

Let the conical surface have half-angle, α. The distance to the cone apex is R_o, while the distances R_1 and R_2 are the distances to the near and far edges of the surface. (For a complete cone $R_1 = R_o$). The impulse response is

$$E_x(t) = \frac{\tan^2\alpha}{8\pi R_o} \left[(R_1 - R_o)\delta\left(t - \frac{2R_1}{c}\right) - (R_2 - R_o)\delta\left(t - \frac{2R_1}{c}\right) - \frac{2}{c}\,\frac{R_o^2}{t^2}\left\{ U\left(t - \frac{2R_1}{c}\right) - U\left(t - \frac{2R_2}{c}\right) \right\} \right]$$

From these simple shapes, results may be obtained for a great many composite shapes.

RADAR SYSTEM

The wideband digital correlator is being combined with a wideband transmitter to make an experimental random signal radar system. The present correlator is being modified to have a 100 Mc bandwidth and it is planned to employ a transmitter power of approximately 20 mw. This will provide adequate performance for initial experimentation which will be limited to ranges of a few hundred meters. Fig. 3 shows a block diagram of the complete random signal radar system. The transmitter portion consists primarily of standard commercially available components. The receiver portion, with the exception of the tunnel diode amplifier and the display unit which are also commercially available components, is essentially complete. It is expected that the system will be operational by the summer of 1966.

PLANS FOR NEXT REPORTING PERIOD

During the next reporting period it is planned to complete assembly and checkout of the radar system and to make some preliminary measurements against simple targets. Analytical work will continue on computing the impulse response of randomly located specularly reflecting targets and a more detailed performance estimate of the random signal radar will be made.

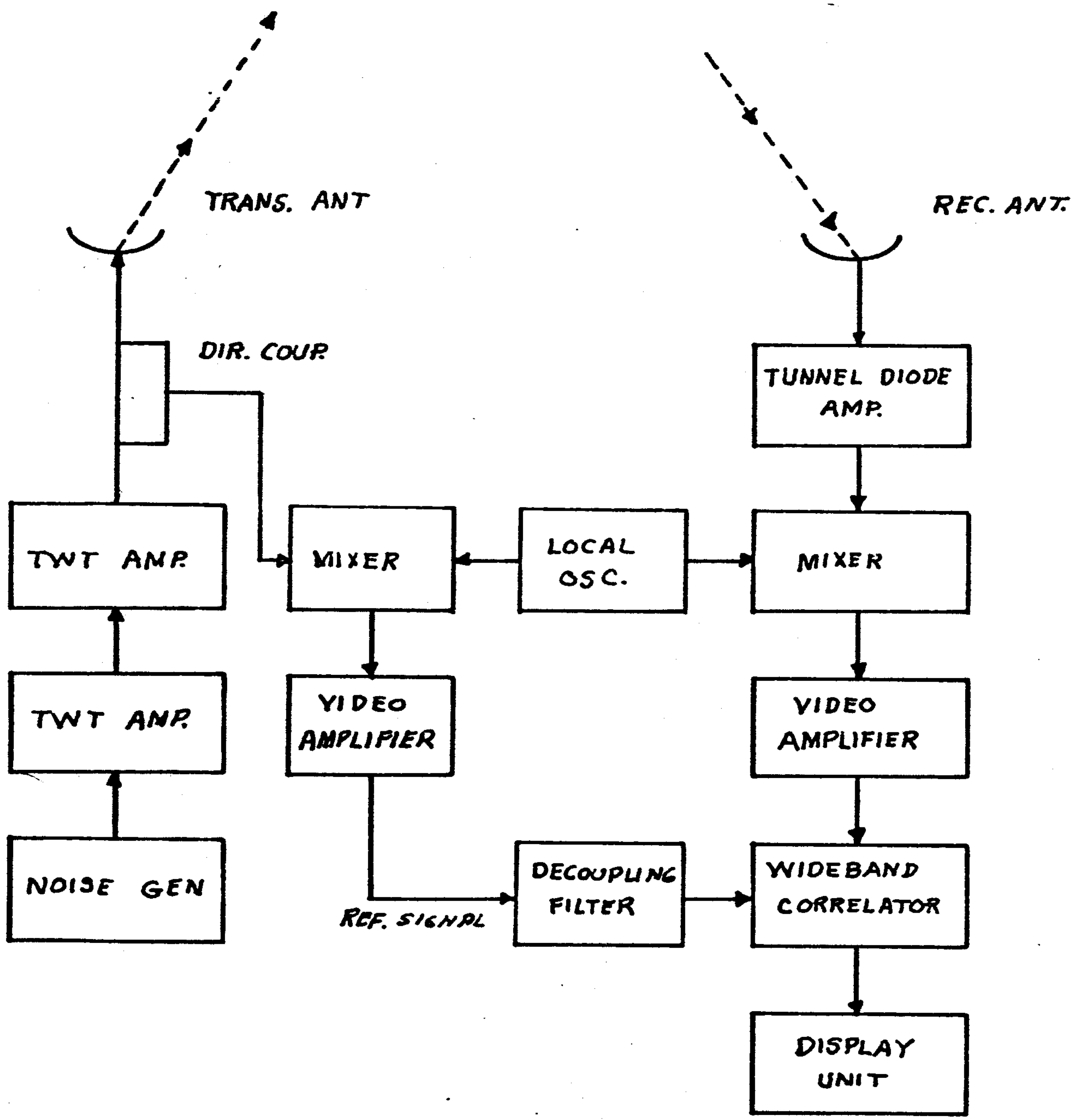


Fig. 3. Random Signal Radar